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A THEORETICAL AND EXPERIMENTAL STUDY OF THE MOBILITY OF LOOSE MATERIALS

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The effect of external and internal parameters of loose materials on their mobility is investigated. A dimensionless equation describing the mobility of loose materials is obtained and confirmed by experimental data.

A loose material is a discrete statistical system of solid particles in contact with one another in a disperse gas medium with or without liquid. The strength of contacts between the particles is determined by their nature and concentration and external factors [1].

During storage of loose materials the colloid-surface interaction between the particles changes, which results in their consolidation. In this case the strength of the contacts between the particles increases and they lose fluidity and can form a monolith. The strength of loose materials increases due to redistribution of fine particles between large ones, which leads to an increase in the contact area between particles, and due to an increase in the intermolecular attraction forces self-adhesion increases, which results a reduction in the mobility of loose material (consolidation).

Consolidation of finely divided loose materials is a process of change in the physicomechanical and physicochemical properties of the materials with time. The most distinctive property of loose materials determining this process is their fluidity or mobility, which is estimated from the ability of loose materials to transfer stress to the enclosing vertical and horizontal surfaces and depends on internal, external, and design parameters of the storage facilities.

The stressed state of the material σ_n, the granulometric composition d_i, and the moisture content W are internal parameters of loose materials.

The air humidity W_1, storage time of loose material B, ambient temperature t, and external effects (vibration, shock, etc.) are external parameters.

According to [2] the mobility of loose materials is estimated by the mobility criterion $m = \sigma_2/\sigma_1$.

The horizontal stress σ_2 arising on the surface of the enclosing walls depends on the vertical stress σ_1, the stress in the dome σ_0, the storage time B of loose material in a prescribed stressed state, the ambient temperature t, the rate of load application, and deformation characteristics, in particular, the bed coefficient e, which is the ratio of the stress to the absolute deformation of the material:

$$e = \frac{\sigma_1}{\delta}.$$

Thus, at a constant ambient temperature $t = \text{const}$, the pressure on the side walls of the container can be written as

$$\sigma_2 = f(\sigma_1, \sigma_0, B, V, e). \tag{1}$$

The apparatus of dimensional analysis can be used as mathematical tools for describing the horizontal transfer of pressure as a function of the aforementioned internal and external parameters of loose materials [3–5].

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According to dimensional analysis the obtained data processed in a suitable dimensionless form that describes the mechanism of pressure transfer to the side walls of the container can be extrapolated. The mechanism characterizes changes in the lateral pressure of a loose material as a function of the aforementioned parameters.

Assuming that in a known range the variables are related by a certain functional relation, we can rewrite Eq. (1) in the following form [3]:

$$\sigma_2 = [\sigma_1]^a [\sigma_0]^z [B]^l [V]^b [e]^r . \tag{2}$$

The equation of dimensionsality will be written in the form

$$\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right] = \left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]^a \left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]^z \left[\mathrm{sec}\right]^l \left[\frac{\mathrm{m}}{\mathrm{sec}^2}\right]^b \left[\frac{\mathrm{N}}{\mathrm{m}^3}\right]^r . \tag{3}$$

A comparison of the exponents of the same units of measurement in the left- and right-hand sides of Eq. (3) gives a system of three equations with five unknown quantities:

$$1 = a + z + r , \quad -2 = -2a - 2z + b - 3r , \quad 0 = l - b . \tag{4}$$

From the π-theorem it is found that for the number $N = 6$ of physical quantities expressed in three basic SI units of measurement the similarity equation should consist of three generalized variables: $\pi = N - n = 6 - 3 = 3$. Consequently, system (4) is solved for the three exponents a, l, b, assuming z and r to be prescribed:

$$a = 1 - z - r , \quad b = l , \quad r = b . \tag{5}$$

Substitution of the values of the exponents a, l, and b from (5) into Eq. (2) gives

$$\sigma_2 = [\sigma_1]^{(1-z-r)} [\sigma_0]^z [B]^r [V]^r [e]^r . \tag{6}$$

Having grouped together the quantities with the same exponent, we can express Eq. (6) in the form

$$\frac{\sigma_2}{\sigma_1} = \left[\frac{\sigma_0}{\sigma_1}\right]^z \left[\frac{BVe}{\sigma_1}\right]^r . \tag{7}$$

where $\sigma_2/\sigma_1 = m$ is the criterion of material mobility; $\sigma_0/\sigma_1 = \kappa$ is the criterion of the ability of the loose material to form a dome; $BV_e/\sigma_1 = \lambda$ is the criterion of storage time for the loose material in the specified stressed state σ_1.

Substitution of the corresponding criteria into Eq. (7) gives the following dimensionless equation:

$$m = \kappa^z \lambda^r . \tag{8}$$

A method for determining the criteria m, κ, and λ was used for experimental verification of Eq. (8) .

A schematic diagram of the device for determining the mobility of materials [6] is shown in Fig. 1a. Rigid housing 1 contains cylinder 2. Upper 4 and lower 5 strain-gauge rings are mounted on three symmetrically located pillars 3. Bottom 6 of the cylinder is screwed into lower ring 5. Loading system 7, containing a rod with piston 8, is located on upper strain-gauge ring 4. The piston is moved by removable handle 9. Rings 4 and 5 serve to measure the vertical force applied to a sample of the test loose material. Sample 10 is located in the cylinder cavity between bottom 6 and piston 8.

On the lower part of the side surface of the cylinder a window was cut out, which is closed with lateral insert 11. During manufacturing the lateral insert is machined together with the cylinder, because of which the cylindrical surface is prevented from being distored. Lateral insert 11 is fastened rigidly to lever 12, which is fixed by special bush 13 with the aid of two axial bearings 14. This suspension system allows the lever to swing with minimum friction losses.

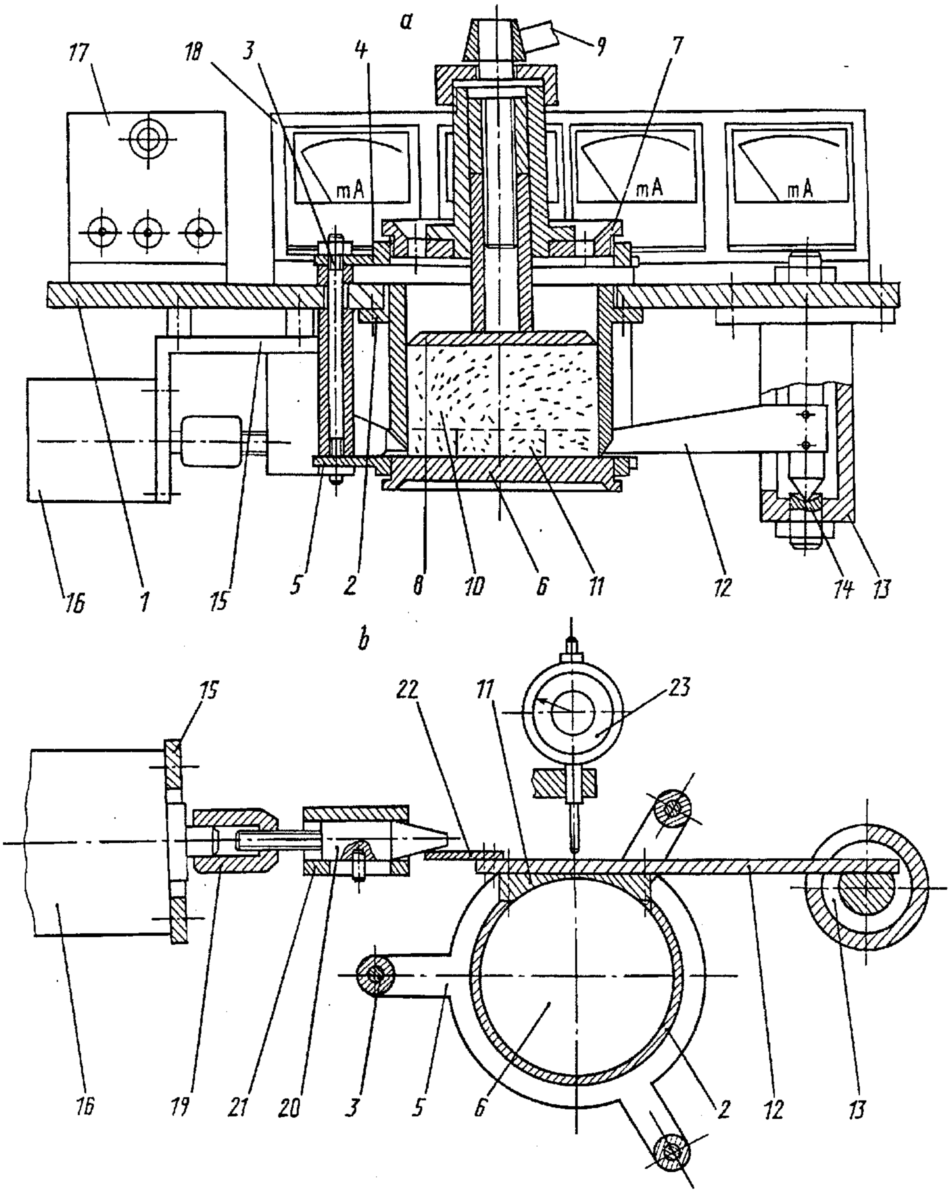

Fig. 1. Schematic diagram of the device for determination of the mobility of loose materials (a) and the system of loading of the lateral insert (b).

The system applying a load to the lateral insert mounted on special bracket 15 and is activated by reversible electric motor 16. Control board 17 of the device and board 18 with recording microammeters are located on the table of the device. The loading system for lateral insert 11 is shown in Fig. 1b. Special bush 19, which holds the tail of loading cone 20, is mounted on the shaft of electric motor 16. The loading cone moves in special housing 21, and its conical part is in contact with strain-gauge beam 22, which is fastened rigidly to the end of swinging lever 12 with lateral insert 11.

Clocktype indicator 23, which is fastened to the housing of the device, functions to record the displacements of the lateral insert, and its leg rests on lever 12.

Strain-gauge beam 22 measures the force pressing lateral insert 11 against cylinder 2.

An additional strain-gauge beam can be used instead of indicator 23. Strain-gauge resistors connected to a strain-gauge amplifier and recording microammeters are glued to the projections of strain-gauge rings 4 and 5 and strain-gauge beam 22.

Experimental Procedure. Before starting the experiments, the device should be thoroughly leveled. Then, by turning handle 9 piston 8 is lifted against the stop and loading system 7 is removed from the device. Then, with the help of electric motor 16 and conical insert 20 the strain-gauge beam is loaded. It presses lateral insert 11

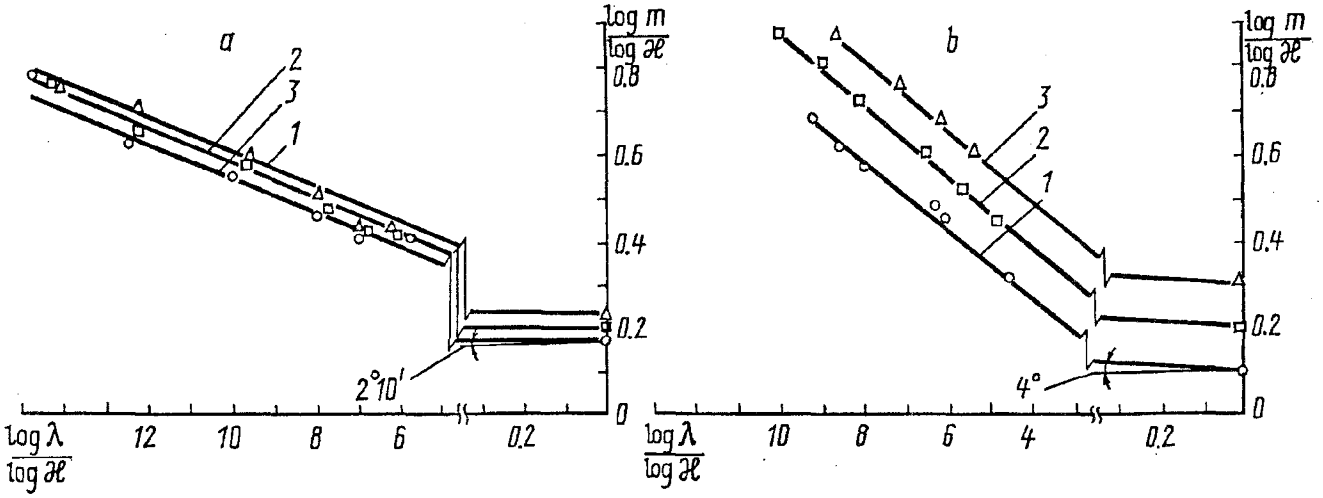

Fig. 2. Plots of $\log m/\log \kappa$ versus $\log \lambda/\log \kappa$ for secondgrade flour at $W =$ 10.8% (a) and powdered cocoa at $W = 4.2\%$ (b): 1) $\sigma_1 = 70$ kPa; 2) 50; 3)

against cylinder 2 with a force exceeding substantially the maximum permissible force of the lateral pressure in the region of the lateral insert of the cylinder.

A certain portion of the loose material is charged into cylinder 2 and its upper layer is thoroughly smoothed out. Then the loading system is installed and indications of the measuring instruments linked with upper 4 and lower 5 strain gauge rings are set to zero.

By turning handle 9 piston 8 is lowered and the material studied is compressed to a prescribed stressed state in the bottom cavity. After the loading process is stabilized, the material is kept under the load for a preset time. Then piston 8 is lifted, the load is removed, and the residual stress σ_0 of the loose material against the bottom is recorded.

Lateral insert 11 is unloaded by electrtic motor 16 and cone 20 in a stepwise manner (within 10%, for the required measuring accuracy) with subsequent loading of the loose material up to a preset vertical stress σ_1. The lateral pressure σ_2 is determined when the pointer of the recording instrument starts to deflect, which corresponds to detachment of the lateral insert.

Thus, while measuring the lateral pressure, the structure of the loose material in the cylinder is not disturbed, since the measurement takes place at the initial moment of detachment of the lateral insert from the cylinder rather than in the fully detached position.

Secondgrade flour with a moisture content $W = 10.8\%$ and powdered cocoa with $W = 4.2\%$ were used as objects of study. The moisture content of the materials was assumed to be that of the air-dried state, i.e., at $\Theta =$ 295 K and an ambient humidity $W_1 = 75 \pm 1\%$. The storage time of the materials varied from 5 min to 5 days. The normal stress varied in the range of 10 to 80 kPa.

The criteria m, λ, and κ were evaluated from experimental data.

In order to determine the criteria of the exponents r and z, the logarithm of Eq. (8) is taken:

$$\log m = r \log \lambda + z \log \kappa . \tag{9}$$

The relations $\log m/\log \kappa = f(\log \lambda/\log \kappa)$ are plotted for the secondgrade flour (Fig. 2a) and the powdered cocoa (Fig. 2b).

It is seen from Fig. 2 that with different storage times of the materials and different stresses σ_1 the experimental points are located so that a straight line can be drawn through them, which cuts off a value equal to the exponent z from the $\log m/\log \kappa$ axis. All these lines obtained at different stresses σ_1 are parallel to one another, i.e., they are inclined at the same angle α, whose tangent is equal to the exponent r. For the secondgrade flour $r = -0.07$ and for the powdered cocoa $r = -0.0437$.

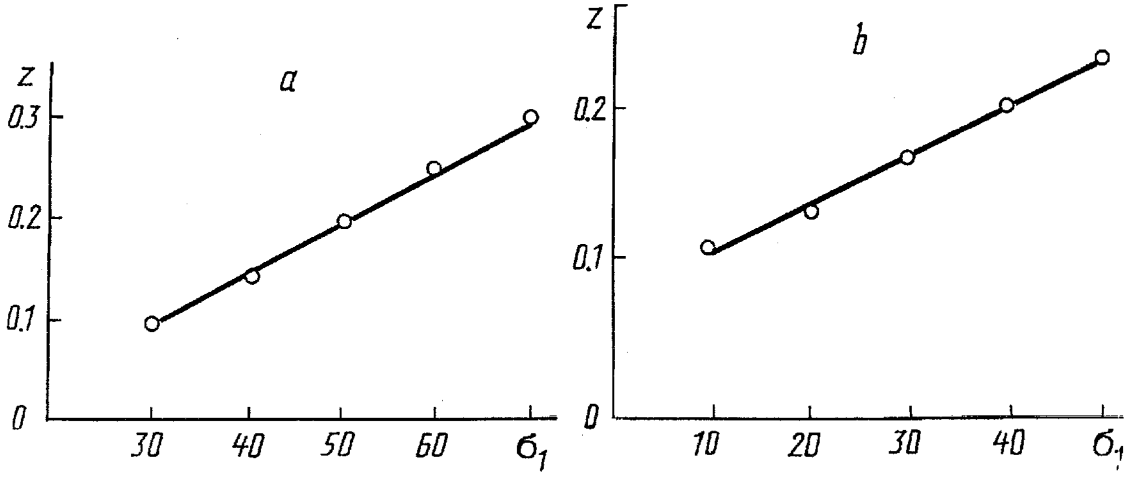

Fig. 3. Plot of the exponent z versus the stress σ_1 for secondgrade flour (a) and powdered cocoa (b).

From the data shown in Fig. 2 it is seen that the exponent z depends on the stressed state σ_1 of the material. A plot of z versus σ_1 is shown in Fig. 3.

Having processed the experimental data $z = f(\sigma_1)$, we obtain the following linear relations: for the secondgrade flour $z = 0.0037\sigma_1 + 0.04$ and for the powdered cocoa $z = 0.005\sigma_1 - 0.06$.

The present mathematical models can be used to determine the criterion of mobility m of loose materials as a function of the storage time λ and the criterion of dome formation κ with an accuracy sufficient for practical purposes.

The mobility criterion can be used in calculations of silos, bunkers, and other storage facilities used in the food, mining, and other industries.

NOTATION

σ_1, vertical stress; σ_2, horizontal stress; σ_0, stress in the dome; B, storage time of loose material; e, bed coefficient; t, ambient temperature; V, rate of loading; δ, absolute deformation of material; W, moisture content of material; W_1, air humidity; a, b, l, z, r, exponents; m, mobility criterion; κ, criterion of the ability of loose material to form a dome; λ, criterion of storage time of loose material under a preset stress.

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